

Hong Kong Mathematics Olympiad (2010 / 2011)

Heat Event (Individual)

香港數學競賽 (2010 / 2011)

初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 求  $2^{2011}$  除以 13 的餘數。

Find the remainder when  $2^{2011}$  is divided by 13.

2. 已知  $x^2 + y^2 = 1$ ，求  $2x + 5y^2$  的極大值。

Given that  $x^2 + y^2 = 1$ , find the maximum value of  $2x + 5y^2$ .

3. 已知  $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$  及  $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$ ，求  $ab$  的值。(答案以根式表示)

Given that  $a + b = \sqrt{\sqrt{2011} + \sqrt{2010}}$  and  $a - b = \sqrt{\sqrt{2011} - \sqrt{2010}}$ , find the value of  $ab$ . (Give your answer in surd form)

4. 在  $\triangle ABC$  內，分別垂直於三條邊  $AB$ 、 $BC$  及  $CA$  的高的比是  $3 : 4 : 5$ 。若三條邊的長均為整數，求  $AB$  的最小值。

In  $\triangle ABC$ , the ratio of the altitudes perpendicular to the three sides  $AB$ ,  $BC$  and  $CA$  is  $3 : 4 : 5$ . If the lengths of the three sides are integers, find the minimum value of  $AB$ .

5. 整數  $x$  減去 12 後是一個整數的平方。將  $x$  加上 19 後則是另一個整數的平方。求  $x$  的值。

An integer  $x$  minus 12 is the square of an integer.  $x$  plus 19 is the square of another integer. Find the value of  $x$ .

6. 甲、乙及丙三人互相傳球。甲首先將球傳出。有多少不同方案使得經過 5 次傳球後，球會回傳給甲？

$A$ ,  $B$  and  $C$  pass a ball among themselves.  $A$  is the first one to pass the ball to the other one. In how many ways will the ball be passed back to  $A$  after 5 passes?

7. 求  $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$  的值。

Find the value of  $\sqrt{7-\sqrt{12}-\sqrt{13-2\sqrt{12}}}$ .

8. 學校推出每張面值為 \$10、\$15、\$25 及 \$40 的四種賣物券。甲班用若干張 \$100 紙幣買了 30 張賣物券，包括其中兩種賣物券各 5 張及另外兩種賣物券各 10 張。問甲班共用了多少張 \$100 紙幣購買賣物券？

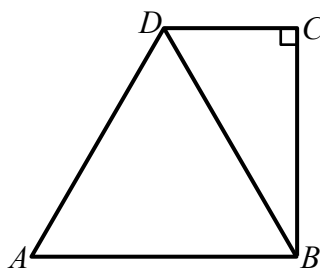
A school issues 4 types of raffle tickets with face values \$10, \$15, \$25 and \$40. Class A uses several one-hundred dollar notes to buy 30 raffle tickets, including 5 tickets each for two of the types and 10 tickets each for the other two types. How many one-hundred dollar notes does Class A use to buy the raffle tickets?

9. 某長方形的長和闊均為整數。若面積比周界大 9，求周界的值。

The length and the width of a rectangle are integers. If its area is larger than its perimeter by 9, find the perimeter.

10. 如圖二， $ABCD$  為一個梯形，其中  $\angle C = 90^\circ$ 。若等邊三角形  $ABD$  的面積為  $16\sqrt{3}$ ，求梯形  $ABCD$  的面積。

In Figure 2,  $ABCD$  is a trapezium with  $\angle C = 90^\circ$ . If the area of the equilateral triangle  $ABD$  is  $16\sqrt{3}$ , find the area of trapezium  $ABCD$ .



圖二

Figure 2